NASA CR-181661

AN EXACT STRATEGY FOR FLEXIBLE WALL ADJUSTMENTS IN TWO-DIMENSIONAL COMPRESSIBLE FLOW

Michael Judd

March 1981

(NASA-CR-181661) AN EXACT STRATEGY FOR FLEXIBLE WALL ADJUSTMENTS IN TWO-DIMENSIONAL COMPRESSIBLE FLOW (Southampton Univ.) 38 p

N88-70831

Unclas
JJ/02 0141228

AN EXACT STRATEGY FOR FLEXIBLE WALL ADJUSTMENTS IN TWO-DIMENSIONAL COMPRESSIBLE FLOW

Michael Judd

March 1981

This report was prepared under contract to The University of Southampton in support of NASA Grant NSG-7172 entitled "The Self-Streamlining of the Test Section of a Transonic Wind Tunnel". The Principal Investigator is Dr. M. J. Goodyer.

An Exact Strategy for Flexible Wall Adjustment in Two Dimensional Compressible Flow.

1. Introduction.

In section 2, a strategy is presented for the adjustment of the top & bottom walls of a two-dimensional tunnel. The strategy is exact within the limits of the theory and no iterative wall movements should be necessary except for effects of physical phenomena not taken into account in the theory. These limits are:-

(1) the theory is linearised and no second order effects are included i.e. wall movements must be small.

(2) compressibility is incorporated in the form of the Prandtl-Glauert factor & no account is taken of transonic flow effects.

(3) no account is taken of the viscous effects in the flow, in particular of any movement in separation or transition points on the model or walls when a wall is moved in or out.

It is anticipated that the effects in (1) and (2) will not be large for moderate model sizes and Mach numbers not too close to unity. Effect (3) may result, for some cases,

in an iteration. However, the strategy presented here does offer a strong possibility of greatly reduced adjustment times for the wall correction procedure.

The detailed analysis leading to the strategy is developed equentially through Appendices A to E.

2. Strategy for Exact Wall Adjustment in a Form Suitable for Programming.

The analysis developed in the Appendices applies to the arrangement shown in fig. 2.1.

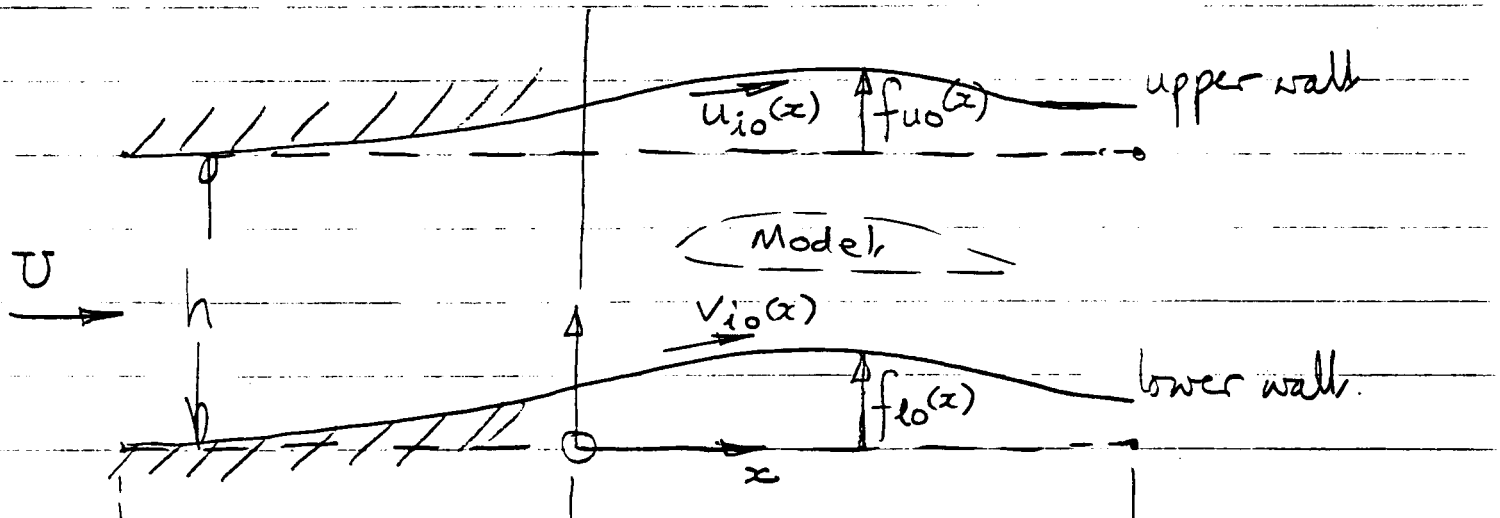


Fig. 2.1

Here U is the undisturbed tunnel velocity, h is the tunnel height between straight walls and 2L is the

overall length of the flexible part of the tunnel working section. For the initial condition shown in fig. 2.1. the upper and lower walls have known wall settings $f_{uo}(x)$ and $f_{lo}(x)$ respectively, when measured from the straight wall positions. With the model present, these settings result in streamwise perturbation velocity components $u_{io}(x)$ and $v_{io}(x)$ when measured from the upper and lower wall static pressure distributions. The new wall positions are $f_u(x)$ and $f_l(x)$, measured respectively from the straight wall positions where :-

Term number

$$f_u(x) = \frac{1}{2} f_{uo}(x) \qquad ①$$

$$- \frac{\beta}{2\pi U} \int_{-L}^{+L} u_{io}(x') \ln \left| \frac{x'-x}{x'+L} \right| dx' \qquad ②$$

$$+ \frac{\beta}{4\pi U} \int_{-L}^{+L} v_{io}(x') \ln \left[\frac{(\beta h)^2 + (x'-x)^2}{(\beta h)^2 + (x'+L)^2} \right] dx' \qquad ③$$

$$- \frac{\beta h}{2\pi} \int_{-L}^{+L} f_{lo}(x') \frac{dx'}{\left[(\beta h)^2 + (x'-x)^2 \right]} \qquad ④$$

This is derived from equation (E2) in Appendix E. Here β is given by $\sqrt{1-M^2}$ where M is the freestream Mach number. Term ① is straightforward. Terms ③ and ④ are the cross-feed inputs from the lower wall velocity and setting.

They are easy to integrate numerically using trapezoidal area contributions, because the integrals have no singularities or irregularities. Term ② requires care as it must be interpreted in the principal value sense (hence the $\oint$ symbol) This means that it has a singularity at $x' = x$ and integration in this region must be carried out analytically with $u_{i0}(x)$ expressed locally as a polynomial. Care must also be taken to ensure that the value of u_{i0} is numerically zero as x' tends to $-L$ i.e. at the upstream end of the flexible section.

The lower wall setting is given by equation (E3) as :-

$$f_1(x) = \frac{1}{2} f_{10}(x)$$

$$+ \frac{\beta}{2\pi U} \oint_{-L}^{+L} V_{10}(x') \ln \left| \frac{x'-x}{x'+L} \right| dx'$$

$$+ \frac{\beta}{4\pi U} \int_{-L}^{+L} u_{i0}(x') \ln \left[\frac{(\beta h)^2 + (x'-x)^2}{(\beta h)^2 + (x'+L)^2} \right] dx'$$

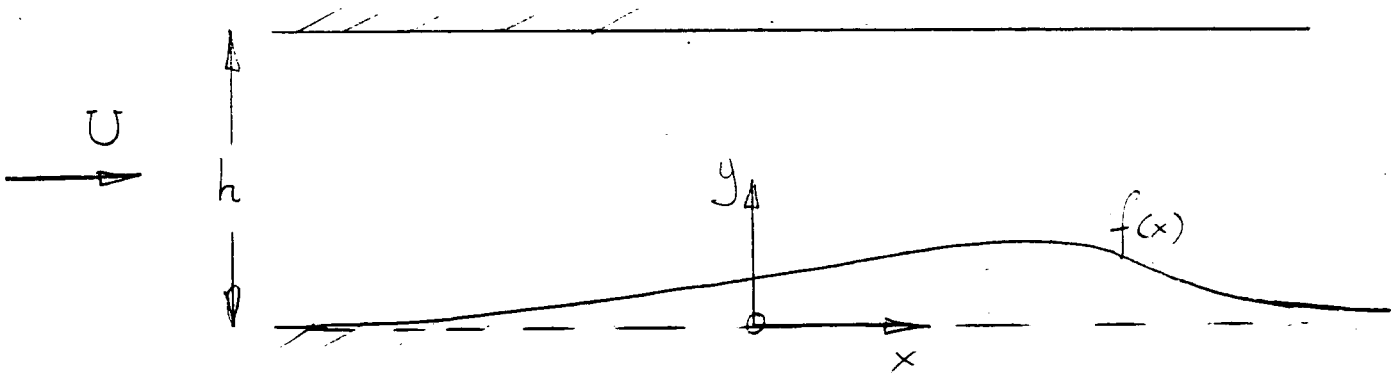
$$- \frac{\beta}{2\pi} \int_{-L}^{+L} f_{u0}(x') \frac{dx'}{\left[(\beta h)^2 + (x'-x)^2 \right]}$$

APPENDIX A. The Fundamental Incompressible Flow Solution:

The effect on one wall of deflection of the second wall in a two-dimensional wind tunnel.

Assumptions.

1. Two-dimensional, parallel walls extending to infinity upstream & downstream. (The walls are finally given finite length.)
2. One wall is given a small distortion from its initial straight form. The shape is defined by $y = f(x)$.
3. The wall distortion tends to zero upstream and down. i.e. $f(x)$ and its derivatives are zero for $x = \pm\infty$. This can be eventually relaxed downstream.



The undisturbed flow velocity is U

The perturbation velocity potential for the disturbed flow is ϕ, and is governed by :-

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \text{—(A1)}$$

The required approximate boundary conditions are :-

zero normal velocity component at upper and lower walls

i.e. $$\frac{\partial \phi}{\partial y} - U \frac{df}{dx} = 0 \qquad \text{at} \quad y = 0 \qquad \text{—(A2)}$$

$$\frac{\partial \phi}{\partial y} = 0 \qquad \text{at} \quad y = h \qquad \text{—(A3)}$$

Define the Fourier transform pair :-

$$\Phi(\omega, y) = \int_{-\infty}^{+\infty} \phi(x, y)\, e^{i\omega x}\, dx \qquad \text{— (A4)}$$

$$\phi(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi(\omega, y)\, e^{-i\omega x}\, d\omega \qquad \text{— (A5)}$$

The Fourier transform of equations (A1), (A2) and (A3) gives :-

$$\frac{d^2\Phi}{dy^2} - \omega^2 \Phi = 0 \qquad \text{— (A6)}$$

$$\frac{d\Phi}{dy} = U F(\omega) \qquad \text{at} \quad y = 0 \qquad \text{— (A7)}$$

$$\frac{d\Phi}{dy} = 0 \qquad \text{at} \quad y = h \qquad \text{— (A8)}$$

where $$F(\omega) = \int_{-\infty}^{+\infty} \frac{df}{dx}\, e^{i\omega x}\, dx = -i\omega \int_{-\infty}^{+\infty} f(x)\, e^{i\omega x}\, dx \qquad \text{— (A9)}$$

The general solution to equation (A6) is

$$\bar{\Phi}(y) = A(\omega)\, e^{\omega y} + B(\omega) e^{-\omega y} \qquad \text{—— (A10)}$$

where $A(\omega)$ and $B(\omega)$ are constants of integration. $A(\omega)$ and $B(\omega)$ are determined using the conditions (A7) and (A8). Finally a solution for $\phi(x,y)$ can be obtained by taking the inverse transform of $\bar{\Phi}(y)$.

$$\phi(x,y) = \frac{iU}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\xi)\, e^{i\omega(\xi - x)} \frac{\cosh \omega(y-h)}{\sinh \omega h}\, d\xi\, d\omega \qquad \text{— (A11)}$$

The u-velocity components at the lower and upper walls can be found from $\frac{\partial \phi}{\partial x}$ at $y = 0$ and $y = h$:-

at the lower wall $(y = 0)$

$$\frac{\partial \phi}{\partial x} = -\frac{\pi U}{4h^2} \int_{-\infty}^{+\infty} f(\xi)\, \operatorname{csch}^2 \frac{\pi(\xi - x)}{2h}\, d\xi$$

and at the upper wall, $(y = h)$

$$\frac{\partial \phi}{\partial x} = \frac{\pi U}{4h^2} \int_{-\infty}^{+\infty} f(\xi)\, \operatorname{sech}^2 \frac{\pi(\xi - x)}{2h}\, d\xi.$$

These may be reduced by an integration by parts to :-

$$\frac{\partial \phi}{\partial x} = -\frac{U}{2h}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \coth\frac{\pi(\xi - x)}{2h}\, d\xi \qquad \text{at } y = 0$$

—(A12)

$$\frac{\partial \phi}{\partial x} = -\frac{U}{2h}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \tanh\frac{\pi(\xi - x)}{2h}\, d\xi \qquad \text{at } y = h$$

—(A13)

In the limit that there is no upper wall ($h \to \infty$), these reduce to :-

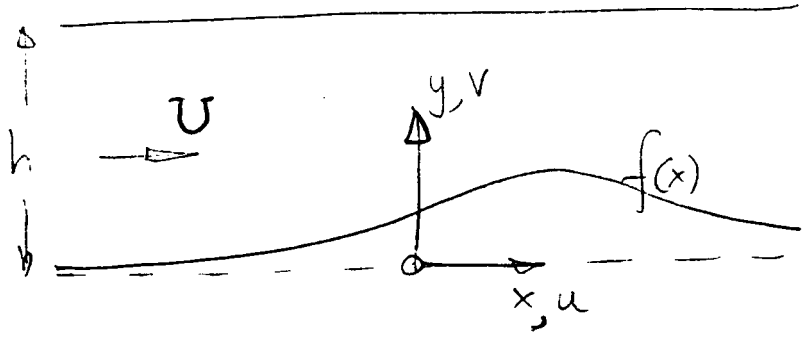
$$\frac{\partial \phi}{\partial x} = -\frac{U}{\pi}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \frac{d\xi}{(\xi - x)} \qquad \text{at } y = 0 \qquad \text{—(A 14)}$$

and $\frac{\partial \phi}{\partial x} \to 0$ as $y \to \infty$.

Equation (A14) is the form that is used in the existing wall movement strategy. Equations (A12) and (A13) are the exact solutions, but they are more complicated to use than the approximate cross-feed method.

APPENDIX B. Compressible Flow Transformation.

The solutions for velocities and velocity potential in Appendix A can be used to give the compressible flow results as follows:



$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \beta = \sqrt{1 - M^2}$$

$$u = \frac{\partial \phi}{\partial x}$$

$$v = \frac{\partial \phi}{\partial y}$$

$$\frac{\partial \phi}{\partial y} - U \frac{df}{dx} = 0 \qquad \text{at} \quad y = 0$$

$$\frac{\partial \phi}{\partial y} = 0 \qquad \text{at} \quad y = h$$

These can be transformed as follows:-

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \text{—(B1)}$$

$$u = \frac{\partial \phi}{\partial x} \qquad \bar{v} = \frac{\partial \phi}{\partial \bar{y}}$$

$$\frac{\partial \phi}{\partial \bar{y}} - \bar{U} \frac{df}{dx} = 0 \quad \text{at } \bar{y} = 0 \qquad \text{—(B2)}$$

$$\frac{\partial \phi}{\partial \bar{y}} = 0 \quad \text{at } \bar{y} = \bar{h} \qquad \text{— (B3)}$$

where $\bar{y} = \beta y$

$$\bar{v} = \frac{v}{\beta}$$

$$\bar{U} = \frac{U}{\beta}$$

$$\bar{h} = \beta h$$

The equations are now in the incompressible form with u, ϕ and x unchanged.

The solutions can be obtained directly from equations (A11), (A12), (A13) and (A14) to give :-

$$\phi(x,\bar{y}) = \frac{i\,\bar{U}}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\xi)\, e^{i\omega(\xi - x)}\, \frac{\cosh\omega(\bar{y}-\bar{h})}{\sinh\omega\bar{h}}\, d\xi\, d\omega \qquad -(B4)$$

or

$$\phi(x,y) = \frac{i\,U}{2\pi\beta}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\xi)\, e^{i\omega(\xi - x)}\, \frac{\cosh\beta\omega(y-h)}{\sinh\beta\omega h}\, d\xi\, d\omega \qquad -(B5)$$

$$\frac{\partial\phi}{\partial x} = -\frac{\pi U}{4\beta^3 h^2}\int_{-\infty}^{+\infty} f(\xi)\, \operatorname{cosech}^2\frac{\pi(\xi - x)}{2\beta h}\, d\xi \qquad \text{at } y = 0 \qquad -(B6)$$

$$\frac{\partial\phi}{\partial x} = \frac{\pi U}{4\beta^3 h^2}\int_{-\infty}^{+\infty} f(\xi)\, \operatorname{sech}^2\frac{\pi(\xi - x)}{2\beta h}\, d\xi \qquad \text{at } y = h. \qquad -(B7)$$

or $$\frac{\partial \phi}{\partial x} = -\frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \coth \frac{\pi(\xi - x)}{2\beta h}\, d\xi \qquad \text{at } y = 0 \qquad -(B8)$$

$$\frac{\partial \phi}{\partial x} = -\frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \tanh \frac{\pi(\xi - x)}{2\beta h}\, d\xi \qquad \text{at } y = h \qquad -(B9)$$

$$u(x,y) = -\frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \frac{\sinh \frac{\pi(\xi - x)}{\beta h}}{\left\{\cosh \frac{\pi(\xi - x)}{\beta h} + \cos \frac{\pi(y-h)}{h}\right\}}\, d\xi \qquad -(B10)$$

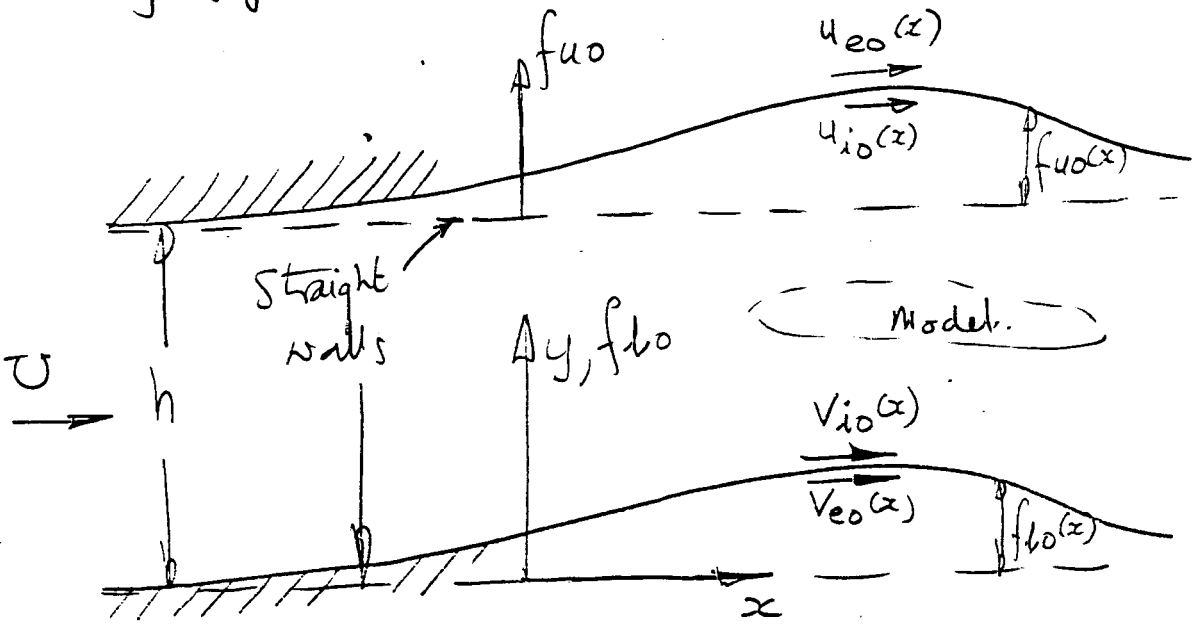
In the limit $h \to \infty$

$$\frac{\partial \phi}{\partial x} = -\frac{U}{\pi\beta}\int_{-\infty}^{+\infty} \frac{df}{d\xi} \frac{d\xi}{(\xi - x)} \qquad \text{at } y = 0 \qquad -(B11)$$

and $\frac{\partial \phi}{\partial x} \to 0$ as $y \to \infty$

APPENDIX C. Application to Wall Movement of the Fundamental Solutions obtained in Appendices A & B.

The starting point is a tunnel configuration with initial upper wall deflection $f_{uo}(x)$ and lower wall deflection $f_{lo}(x)$. These are measured positive upwards from the straight wall condition as shown in the following figure :-



NOTE THAT $f_{uo}(x)$ AND $f_{lo}(x)$ ARE THE GIVEN INITIAL WALL SETTINGS.

$u_{io}(x)$ AND $V_{io}(x)$ ARE THE MEASURED INCREMENTS IN STREAMWISE VELOCITY AT THE UPPER AND LOWER WALLS RESPECTIVELY WITH THE MODEL PRESENT, OBTAINED FROM THE WALL STATIC PRESSURE DISTRIBUTIONS WITH THE WALL SETTINGS $f_{uo}(x)$ AND $f_{lo}(x)$.

$u_{eo}(x)$ and $v_{eo}(x)$ are the (imaginary) external wall velocity components resulting from the settings $f_{uo}(x)$ and $f_{lo}(x)$. Equation (B11) can be used to give :-

$$u_{eo}(x) = -\frac{U}{\pi\beta}\int_{-\infty}^{+\infty} \frac{df_{uo}}{d\xi}\,\frac{d\xi}{(\xi - x)}$$

$$\text{and } v_{eo}(x) = +\frac{U}{\pi\beta}\int_{-\infty}^{+\infty} \frac{df_{lo}}{d\xi}\,\frac{d\xi}{(\xi - x)}$$

In fact these equations are not subsequently required. However, because of the model presence we have

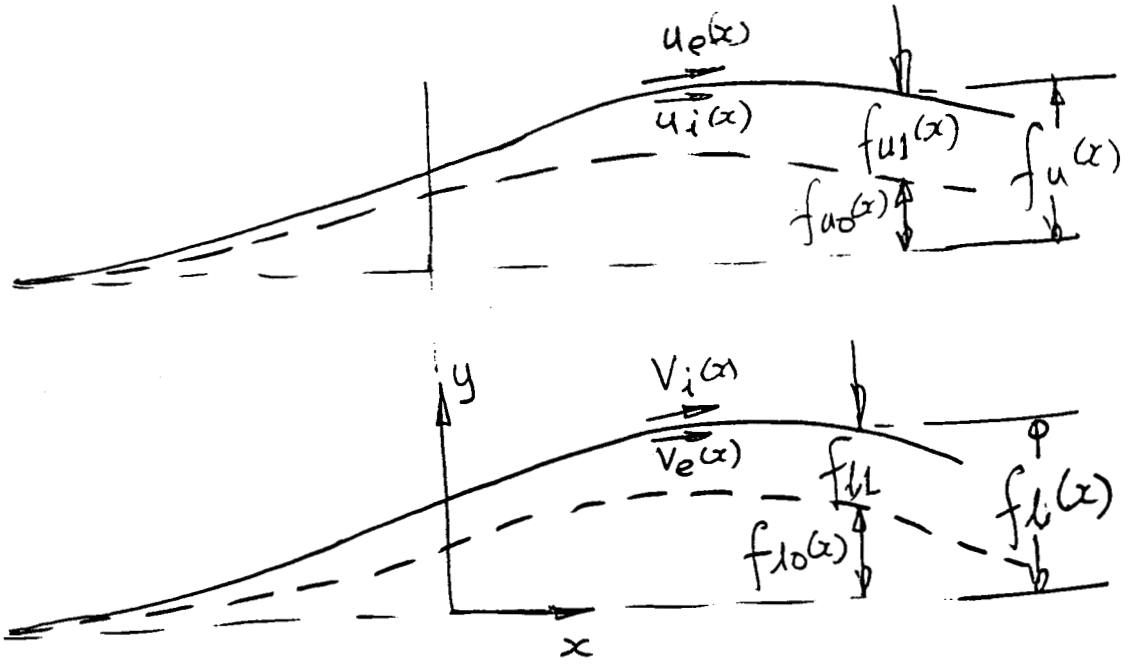
$$u_{eo}(x) \neq u_{io}(x)$$

$$v_{eo}(x) \neq v_{io}(x)$$

The wall positions must therefore be moved from $f_{uo}(x)$ and $f_{lo}(x)$ to new values, $f_u(x)$ and $f_l(x)$, so that, at these new positions, the measured internal wall velocity distributions are identical to the calculated external ones.

The analysis of Appendix B can be used to predict exactly (within the limits of the theory) the new values of $f_u(x)$ and $f_l(x)$ without the need for an iterative procedure.

The method uses the following notation :-



$$f_u(x) = f_{u0}(x) + f_{u1}(x) \qquad \text{—(C1)}$$

$$f_l(x) = f_{l0}(x) + f_{l1}(x) \qquad \text{— (C2)}$$

where $f_{uL}(x)$ and $f_{l1}(x)$ are <u>incremental changes in wall positions</u>.

Using equation (B 11), we obtain :—

$$u_e(x) = -\frac{U}{\pi\beta}\int_{-\infty}^{+\infty} \frac{df_u}{d\xi}\,\frac{d\xi}{(\xi - x)} \qquad \text{— (C3)}$$

$$V_e(x) = \frac{U}{\pi\beta}\int_{-\infty}^{+\infty} \frac{df_l}{d\xi}\,\frac{d\xi}{(\xi - x)} \qquad \text{— (C4)}$$

Using equations (B 8) and (B9), we obtain :—

$$u_i(x) = u_{io}(x) + \frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df_{u1}}{d\xi} \coth\frac{\pi(\xi-x)}{2\beta h}\, d\xi$$

$$- \frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df_{u1}}{d\xi} \tanh\frac{\pi(\xi-x)}{2\beta h}\, d\xi \qquad \text{— (C5)}$$

and

$$V_i(x) = V_{io}(x) + \frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df_{u1}}{d\xi} \tanh\frac{\pi(\xi-x)}{2\beta h}\, d\xi$$

$$- \frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty} \frac{df_{11}}{d\xi} \coth\frac{\pi(\xi-x)}{2\beta h}\, d\xi \qquad \text{— (C6)}$$

The mathematical formulation for the wall streamline condition is

$$u_e(x) = u_i(x) \qquad \text{— (C7)}$$

and $$V_e(x) = V_i(x) \qquad \text{— (C8)}$$

Substitution for u_e, u_i, V_e and V_i in equations (C7) and (C8) using equations (C3), (C4), (C5) and (C6) gives the following :—

$$-\frac{U}{\pi\beta}\int_{-\infty}^{+\infty}\frac{df_{u0}}{d\xi}\frac{d\xi}{(\xi-x)} - \frac{U}{\pi\beta}\int_{-\infty}^{+\infty}\frac{df_{u1}}{d\xi}\frac{d\xi}{(\xi-x)}$$

$$= u_{i0}(x) + \frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty}\frac{df_{u1}}{d\xi}\coth\frac{\pi(\xi-x)}{2\beta h}\,d\xi$$

$$-\frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty}\frac{df_{l1}}{d\xi}\tanh\frac{\pi(\xi-x)}{2\beta h}\,d\xi \qquad -(C9)$$

and

$$\frac{U}{\pi\beta}\int_{-\infty}^{+\infty}\frac{df_{l0}}{d\xi}\frac{d\xi}{(\xi-x)} + \frac{U}{\pi\beta}\int_{-\infty}^{+\infty}\frac{df_{l1}}{d\xi}\frac{d\xi}{(\xi-x)}$$

$$= V_{i0}(x) + \frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty}\frac{df_{u1}}{d\xi}\tanh\frac{\pi(\xi-x)}{2\beta h}\,d\xi$$

$$-\frac{U}{2\beta^2 h}\int_{-\infty}^{+\infty}\frac{df_{l1}}{d\xi}\coth\frac{\pi(\xi-x)}{2\beta h}\,d\xi \qquad -(C10)$$

The following shorthand is introduced :-

$$g_{u0}(\xi) \equiv \frac{df_{u0}(\xi)}{d\xi} \qquad g_{u1}(\xi) = \frac{df_{u1}}{d\xi}$$

$$g_{l0}(\xi) = \frac{df_{l0}}{d\xi} \qquad g_{l1}(\xi) = \frac{df_{l1}}{d\xi}$$

Also let

$$\bar{u}_0(x) = \frac{\beta u_{i0}(x)}{U}$$

and $$\bar{v}_0(x) = \frac{\beta v_{i0}(x)}{U}$$

Then equations (C9) and (C10) can be rearranged in the forms :—

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} g_{u1}(\xi)\left\{\frac{1}{(\xi-x)} + \frac{\pi}{2\beta h}\coth\frac{\pi(\xi-x)}{2\beta h}\right\}d\xi - \frac{1}{2\beta h}\int_{-\infty}^{+\infty} g_{l1}(\xi)\tanh\frac{\pi(\xi-x)}{2\beta h}\,d\xi$$

$$= -\bar{u}_0(x) - \frac{1}{\pi}\int_{-\infty}^{+\infty} g_{u0}(\xi)\,\frac{d\xi}{(\xi-x)} \qquad \text{— (C11)}$$

and

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} g_{l1}(\xi)\left\{\frac{1}{(\xi-x)} + \frac{\pi}{2\beta h}\coth\frac{\pi(\xi-x)}{2\beta h}\right\}d\xi - \frac{1}{2\beta h}\int_{-\infty}^{+\infty} g_{u1}(\xi)\tanh\frac{\pi(\xi-x)}{2\beta h}\,d\xi$$

$$= \bar{v}_0(x) - \frac{1}{\pi}\int_{-\infty}^{+\infty} g_{l0}(\xi)\,\frac{d\xi}{(\xi-x)} \qquad \text{— (C12)}$$

APPENDIX D. Modified Form of Wall Movement Equations and Their Inversion using Fourier Component Representations of the Wall Settings.

Various shorthands (as defined on the next page) are used to reduce equations (C 11) and (C 12) to the forms (D 1) and (D 2). By addition + subtraction these equations are reduced to the more symmetric and anti-symmetric forms given by (D4) and (D5) on page D3.

The remainder of the Appendix is devoted to the inversion of the integral equations (D4) and (D5) to give the desired results of equations (D15) on page D 9 and (D25) on page D 13.

ei $\xi = \frac{\pi\xi}{2\beta h}$ and $X = \frac{\pi x}{2\beta h}$.

Equations (C11) and (C12) can then be reduced to :—

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} g_{u1}(\xi)\ K_1(\xi - X)\,d\xi - \frac{1}{\pi}\int_{-\infty}^{+\infty} g_{l1}(\xi)\ K_2(\xi - X)\,d\xi = \bar{U}_0(X) \qquad \text{— (D1)}$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} g_{l1}(\xi)\ K_1(\xi - X)\,d\xi - \frac{1}{\pi}\int_{-\infty}^{+\infty} g_{u1}(\xi)\ K_2(\xi - X)\,d\xi = V_0(X) \qquad \text{— (D2)}$$

where

$$K_1(\xi - X) = \frac{1}{\xi - X} + \operatorname{ctnh}(\xi - X)$$

$$K_2(\xi - X) = \tanh(\xi - X)$$

$$U_0(X) = -\bar{u}_0(X) - \frac{1}{\pi}\int_{-\infty}^{+\infty} g_{u0}(\xi)\ \frac{d\xi}{(\xi - X)}$$

and

$$V_0(X) = \bar{v}_0(X) - \frac{1}{\pi}\int_{-\infty}^{+\infty} g_{l0}(\xi)\ \frac{d\xi}{(\xi - X)}$$

Adding equations (C13) and (C14) gives :—

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} \left[g_{u1}(\xi) + g_{l1}(\xi)\right]\left[K_1(\xi - X) - K_2(\xi - X)\right]d\xi = U_0(X) + V_0(X) \qquad \text{— (D3)}$$

Subtracting them gives:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}\left[g_{u1}(\xi)-g_{l1}(\xi)\right]\left[K_1(\xi-x)+K_2(\xi-x)\right]d\xi = U_0(x)-V_0(x)$$

or

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} G_1(\xi)\,K_3(\xi-x)\,d\xi = F_1(x) \qquad -(D4)$$

and $$\frac{1}{\pi}\int_{-\infty}^{+\infty} G_2(\xi)\,K_4(\xi-x)\,d\xi = F_2(x) \qquad -(D5)$$

where

$$G_1(\xi) = g_{u1}(\xi) + g_{l1}(\xi)$$

$$G_2(\xi) = g_{u1}(\xi) - g_{l1}(\xi)$$

$$F_1(x) = U_0(x) + V_0(x)$$

$$F_2(x) = U_0(x) - V_0(x)$$

and the kernel functions are given by:

$$K_3(\xi-x) = K_1(\xi-x) - K_2(\xi-x)$$

$$= \frac{1}{(\xi-x)} + \frac{2}{\sinh 2(\xi-x)}$$

and: $K_4(\beta - x) = K_1(\beta - x) + K_2(\beta - x)$

$$= \frac{1}{(\beta - x)} + \frac{2}{\tanh 2(\beta - x)}$$

4. Fourier component solution of the equation

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} G_1(\xi)\, K_3(\xi - X)\, d\xi = F_1(X) \quad -(D4)$$

is sought using a substitution of the form :-

$$G_1(\xi) = \int_0^{\infty} \{C_1(\lambda)\cos\lambda\xi + S_1(\lambda)\sin\lambda\xi\}\, d\lambda \quad -(D6)$$

Consider an integral I_1 of the following form :-

$$I_1 = \int_{-\infty}^{+\infty} \cos\lambda\xi \cdot K_3(\xi - x)\, d\xi$$

$$= \int_{-\infty}^{+\infty} \left\{ \frac{\cos\lambda\xi}{(\xi - x)} + \frac{2\cos\lambda\xi}{\sinh 2(\xi - X)} \right\} d\xi$$

$$= -\pi \sin\lambda X + 2\int_{-\infty}^{+\infty} \frac{\cos\lambda(Z+X)}{\sinh 2Z}\, dZ \quad \text{with } Z = \xi - X$$

$$= -\pi\sin\lambda X + 2\cos\lambda X \int_{-\infty}^{+\infty} \frac{\cos\lambda Z}{\sinh 2Z}\, dZ - 2\sin\lambda X \int_{-\infty}^{+\infty} \frac{\sin\lambda Z}{\sinh 2Z}\, dZ$$

$$= -\pi\sin\lambda X - 4\cdot\frac{\pi}{4}\sin\lambda X \tanh\frac{\pi\lambda}{4}$$

Finally $I_1 = -\pi\left(1 + \tanh\frac{\pi\lambda}{4}\right)\sin\lambda X \quad -(D7)$

Consider also an integral I_2 of the following form :-

$$I_2 = \int_{-\infty}^{+\infty} \sin\lambda\xi \; K_3(\xi - x)\, d\xi$$

$$= \int_{-\infty}^{+\infty} \left\{ \frac{\sin\lambda\xi}{(\xi - x)} + \frac{2\sin\lambda\xi}{\sinh 2(\xi - x)} \right\} d\xi$$

$$= \pi \cos\lambda x + 2\int_{-\infty}^{+\infty} \frac{\sin\lambda(z+x)}{\sinh 2z}\, dz$$

$$= \pi\cos\lambda x + 2\cos\lambda x \int_{-\infty}^{+\infty} \frac{\sin\lambda z}{\sinh 2z}\, dz$$

$$+ 2\sin\lambda x \int_{-\infty}^{+\infty} \frac{\cos\lambda z}{\sinh 2z}\, dz$$

$$= \pi\cos\lambda x + 4\cdot\frac{\pi}{4}\cos\lambda x \tanh\frac{\pi\lambda}{4}$$

$$= \pi\left(1 + \tanh\frac{\pi\lambda}{4}\right)\cos\lambda x \qquad \text{— (D8)}$$

If the form for $G_1(\xi)$ in (D6) is substituted in equation (D4) and the results for I_1 and I_2 given by (D7) and (D8) are used, the following result is obtained :—

$$\frac{1}{\pi}\int_0^{\infty} \pi\left(1 + \tanh\frac{\pi\lambda}{4}\right)\left\{S_1(\lambda)\cos\lambda x - C_1(\lambda)\sin\lambda x\right\} d\lambda = F_1(x)$$

— (D9)

Now let $F_1(x) = F_{1c}(x) + F_{1s}(x)$

where $F_{1c}(x)$ is an <u>even function</u> with respect to $X = 0$

and $F_{1s}(x)$ is an <u>odd function</u>

i.e. $F_{1c}(-x) = + F_{1c}(x)$

and $F_{1s}(-x) = - F_{1s}(x)$

$F_{1c}(x)$ and $F_{1s}(x)$ are obtained from $F_1(x)$ using

$$F_{1c}(x) = \frac{1}{2}\left\{F_1(x) + F_1(-x)\right\}$$

and $$F_{1s}(x) = \frac{1}{2}\left\{F_1(x) - F_1(-x)\right\}$$

Thus equation (D9) can be used to give :-

$$F_{1c}(x) = \int_0^\infty \left(1+\tanh\frac{\pi\lambda}{4}\right) S_1(\lambda) \cos\lambda X \, d\lambda \quad \text{— (D 10)}$$

and

$$F_{1s}(x) = -\int_0^\infty \left(1+\tanh\frac{\pi\lambda}{4}\right) C_1(\lambda) \sin\lambda X \, d\lambda \quad \text{— (D 11)}$$

These can be inverted by Fourier transforms to give :-

$$\left(1+\tanh\frac{\pi\lambda}{4}\right) S_1(\lambda) = \frac{2}{\pi}\int_0^\infty F_{1c}(x) \cos\lambda X \, dx \quad \text{— (D 12)}$$

and

$$\left(1+\tanh\frac{\pi\lambda}{4}\right) C_1(\lambda) = -\frac{2}{\pi}\int_0^\infty F_{1s}(x) \sin\lambda X \, dX \quad \text{— (D13)}$$

The results for $S_1(\lambda)$ and $C_1(\lambda)$ can be used in equation (D6) to give :-

$$G_1(\xi) = \frac{2}{\pi}\int_0^\infty \int_0^\infty \frac{\{F_{1C}(x)\sin\lambda\xi\cos\lambda x - F_{1S}(x)\cos\lambda\xi\sin\lambda x\}}{\left(1+\tanh\frac{\pi\lambda}{4}\right)} d\lambda dx$$

$$\text{or } G_1(\xi) = \frac{1}{\pi}\int_0^\infty \int_0^\infty \{F_1(x)\sin\lambda(\xi-x) + F_1(-x)\sin\lambda(\xi+x)\} \frac{d\lambda dx}{\left(1+\tanh\frac{\pi\lambda}{4}\right)} \quad -(D14)$$

when the definitions for F_{1c} and F_{1s} are used.

Consider the integral

$$I_p = \int_0^\infty \frac{\sin p\lambda}{\left(1+\tanh\frac{\pi\lambda}{4}\right)} d\lambda \qquad \text{where } p = \xi \pm x$$

$$\text{Now } \frac{1}{1+\tanh\frac{\pi\lambda}{4}} = \frac{e^{\frac{\pi\lambda}{4}} + e^{-\frac{\pi\lambda}{4}}}{e^{\frac{\pi\lambda}{4}} + e^{-\frac{\pi\lambda}{4}} + e^{\frac{\pi\lambda}{4}} - e^{-\frac{\pi\lambda}{4}}} = \frac{1}{2}\left(1+e^{-\frac{\pi\lambda}{2}}\right)$$

Hence

$$I_p = \frac{1}{2}\int_0^\infty \left(1+e^{-\frac{\pi\lambda}{2}}\right)\sin p\lambda\, d\lambda$$

$$= \frac{1}{2}\int_0^\infty \sin p\lambda\, d\lambda + \frac{1}{2}\cdot\frac{p}{\left(\frac{\pi}{2}\right)^2 + p^2}$$

This result can be used in equation (D14) to obtain :-

$$G_1(\xi) = \frac{1}{2\pi}\int_0^\infty \left\{\frac{(\xi-x)F_1(x)}{\left[\left(\frac{\pi}{2}\right)^2+(\xi-x)^2\right]} + \frac{(\xi+x)F_1(-x)}{\left[\left(\frac{\pi}{2}\right)^2+(\xi+x)^2\right]}\right\} dx$$

$$+ \frac{1}{2\pi}\int_0^\infty \int_0^\infty \{F_1(x)\sin\lambda(\xi-x) + F_1(-x)\sin\lambda(\xi+x)\} d\lambda dx$$

Combining the integral limits gives :—

$$G_1(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{(\vartheta - x)}{\left\{\left(\frac{\pi}{2}\right)^2 + (\vartheta - x)^2\right\}} F_1(x)\, dx$$

$$+ \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_1(x)\left\{\int_0^{\infty} \sin\lambda(\vartheta - x)\, d\lambda\right\} dx$$

Use the definition

$$\int_0^{\infty} \sin\lambda(\vartheta - x)\, d\lambda = \lim_{\mu \to 0} \int_0^{\infty} e^{-\mu\lambda} \sin\lambda(\vartheta - x)\, d\lambda$$

$$= \lim_{\mu \to 0} \frac{(\vartheta - x)}{\mu^2 + (\vartheta - x)^2}$$

$$= \frac{1}{(\vartheta - x)}$$

to obtain the final inverted form of equation (D4) as :—

$$G_1(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left\{\frac{1}{(\vartheta - x)} + \frac{(\vartheta - x)}{\left[\left(\frac{\pi}{2}\right)^2 + (\vartheta - x)^2\right]}\right\} F_1(x)\, dx$$

— (D15)

In a similar way, a Fourier component solution can be sought for the second equation :-

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} G_2(\xi) K_4(\xi - x)\, d\xi = F_2(x) \qquad \text{—— (D5)}$$

Let $G_2(\xi) = \int_0^{\infty} \{ C_2(\lambda)\cos\lambda\xi + S_2(\lambda)\sin\lambda\xi \}\, d\lambda$ — (D16)

Consider an integral of the form :-

$$I_3 = \int_{-\infty}^{+\infty} \cos\lambda\xi \; K_4(\xi - x)\, d\xi$$

$$= \int_{-\infty}^{+\infty} \left\{ \frac{\cos\lambda\xi}{(\xi - x)} + \frac{2\cos\lambda\xi}{\tanh 2(\xi - x)} \right\} d\xi$$

$$= -\pi \sin\lambda x + 2\cos\lambda x \int_{-\infty}^{+\infty} \frac{\cot\lambda z}{\tanh 2z}\, dz$$

$$- 2\sin\lambda x \int_{-\infty}^{+\infty} \frac{\sin\lambda z}{\tanh 2z}\, dz$$

$$= -\pi \sin\lambda x - 4\sin\lambda x \int_0^{\infty} \frac{\sin\lambda z}{\tanh 2z}\, dz$$

$$= -\pi\sin\lambda x - 4\sin\lambda x \int_0^{\infty} \sin\lambda z (\operatorname{ctnh} 2z - 1)\, dz$$

$$- 4\sin\lambda x \lim_{\mu \to 0} \int_0^{\infty} e^{-\mu z} \sin\lambda z\, dz$$

$$= -\pi\left(1 + 4.\frac{1}{4}\operatorname{ctnh}\frac{\pi\lambda}{4} - \frac{4}{\pi\lambda}\right)\sin\lambda x - \lim_{\mu \to 0} \frac{4\lambda}{(\mu^2 + \lambda^2)} \sin\lambda x$$

Finally.

$$I_3 = -\pi\left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right)\sin\lambda X \qquad \text{— (D 17)}$$

Consider also an integral :—

$$I_4 = \int_{-\infty}^{+\infty} \sin\lambda\xi\, K_4(\xi - x)\, d\xi$$

$$= \int_{-\infty}^{+\infty} \left\{ \frac{\sin\lambda\xi}{(\xi - x)} + \frac{2\sin\lambda\xi}{\tanh 2(\xi - x)} \right\} d\xi \qquad \begin{aligned} z &= \xi - x \\ \xi &= z + x \end{aligned}$$

$$= \pi\cos\lambda X + 2\cos\lambda X \int_{-\infty}^{+\infty} \frac{\sin\lambda z}{\tanh 2z}\, dz$$

$$+ 2\sin\lambda X \int_{-\infty}^{+\infty} \frac{\cos\lambda z}{\tanh 2z}\, dz$$

This gives from the I_3 analysis :—

$$I_4 = \pi\left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right)\cos\lambda X \qquad \text{— (D 18)}$$

Substitution for I_3 & I_4 in $G_2(\xi)$ and the initial equation (D5) gives :—

$$\frac{1}{\pi}\int_0^{\infty} \pi\left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right)\left\{S_2(\lambda)\cos\lambda X - C_2(\lambda)\sin\lambda X\right\} d\lambda = F_2(X)$$

$$\int_0^{\infty} \left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right)\left\{S_2(\lambda)\cos\lambda X - C_2(\lambda)\sin\lambda X\right\} d\lambda = F_2(X)$$

— (D 19)

Now let $F_2(x) = F_{2c}(x) + F_{2s}(x)$

where $F_{2c}(x)$ is an <u>even function</u> with respect to $X = 0$

and $F_{2s}(x)$ is an <u>odd function</u>.

i.e. $F_{2c}(-x) = F_{2c}(x)$

and $F_{2s}(-x) = -F_{2s}(x)$

$F_{2c}(x)$ and $F_{2s}(x)$ are obtained from $F_2(x)$ using

$$F_{2c}(x) = \frac{1}{2}\left\{F_2(x) + F_2(-x)\right\}$$

and $$F_{2s}(x) = \frac{1}{2}\left\{F_2(x) - F_2(-x)\right\}$$

Thus equation (D 19) can be used to give :-

$$F_{2c}(x) = \int_0^\infty \left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right) S_2(\lambda) \cos\lambda X \, d\lambda \qquad \text{— (D 20)}$$

and $$F_{2s}(x) = -\int_0^\infty \left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right) C_2(\lambda) \sin\lambda X \, d\lambda \qquad \text{— (D 21)}$$

These can be inverted to give :-

$$\left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right) S_2(\lambda) = \frac{2}{\pi}\int_0^\infty F_{2c}(x) \cos\lambda X \, dX \qquad \text{— (D 22)}$$

and

$$\left(1 + \operatorname{ctnh}\frac{\pi\lambda}{4}\right) C_2(\lambda) = -\frac{2}{\pi}\int_0^\infty F_{2s}(x) \sin\lambda X \, dX \qquad \text{— (D 23)}$$

The results for $S_2(\lambda)$ and $C_2(\lambda)$ can be used in equation (D16) to give :—

$$G_2(\xi) = \frac{2}{\pi}\int_0^\infty\int_0^\infty \left\{F_{2c}(x)\sin\lambda\xi\cos\lambda x - F_{2s}(x)\cos\lambda\xi\sin\lambda x\right\} \frac{d\lambda\, dx}{\left(1+\operatorname{ctnh}\frac{\pi\lambda}{4}\right)}$$

$$\therefore G_2(\xi) = \frac{1}{\pi}\int_0^\infty\int_0^\infty \left\{F_2(x)\sin\lambda(\xi-x) + F_2(-x)\sin\lambda(\xi+x)\right\} \frac{d\lambda\, dx}{\left(1+\operatorname{ctnh}\frac{\pi\lambda}{4}\right)} \quad \text{—(D24)}$$

But $$\frac{1}{1+\operatorname{ctnh}\frac{\pi\lambda}{4}} = \frac{e^{\frac{\pi\lambda}{4}} - e^{-\frac{\pi\lambda}{4}}}{e^{\frac{\pi\lambda}{4}} - e^{-\frac{\pi\lambda}{4}} + e^{\frac{\pi\lambda}{4}} + e^{-\frac{\pi\lambda}{4}}} = \frac{1}{2}\left(1 - e^{-\frac{\pi\lambda}{2}}\right)$$

Hence by comparison with the solution for $G_1(\xi)$

$$G_2(\xi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left\{\frac{1}{(\xi-x)} - \frac{(\xi-x)}{\left[\left(\frac{\pi}{2}\right)^2 + (\xi-x)^2\right]}\right\} F_2(x)\, dx$$

— (D25)

Equations (D15) and (D25) can be written in the forms :—

$$g_{u1}(\vartheta) + g_{l1}(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{1}{(\vartheta-x)} + \frac{(\vartheta-x)}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}\right\}\left\{U_0(x)+V_0(x)\right\}dx$$

and

$$g_{u1}(\vartheta) - g_{l1}(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{1}{(\vartheta-x)} - \frac{(\vartheta-x)}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}\right\}\left\{U_0(x)-V_0(x)\right\}dx$$

By addition and subtraction this gives :—

$$g_{u1}(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{U_0(x)}{(\vartheta-x)} + \frac{(\vartheta-x)\,V_0(x)}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}\right\}dx \quad — (D26)$$

and

$$g_{l1}(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{(\vartheta-x)\,U_0(x)}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]} + \frac{V_0(x)}{(\vartheta-x)}\right\}dx \quad —(D27)$$

APPENDIX E. Reduction of Inverted Equations to Forms Suitable for Computation.

Equation (D26) becomes:-

$$g_{u1}(\vartheta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{U_0(x)}{(\vartheta-x)} + \frac{(\vartheta-x)V_0(x)}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}\right\}dx$$

$$= -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\bar{u}_0(x)\,dx}{(\vartheta-x)}$$

$$-\frac{1}{2\pi^2}\int_{-\infty}^{+\infty}\frac{g_{u0}(\vartheta_1)\,d\vartheta_1}{(\vartheta_1-x)}\int_{-\infty}^{+\infty}\frac{dx}{(\vartheta-x)}$$

$$+\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{(\vartheta-x)\,\bar{v}_0(x)\,dx}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}$$

$$-\frac{1}{2\pi^2}\int_{-\infty}^{+\infty}\frac{g_{l0}(\vartheta_1)}{(\vartheta_1-x)}\,d\vartheta_1\int_{-\infty}^{+\infty}\frac{(\vartheta-x)\,dx}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}$$

$$= -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\bar{u}_0(x)}{(\vartheta-x)}\,dx - \frac{1}{2}g_{u0}(\vartheta)$$

$$+\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{(\vartheta-x)\,\bar{v}_0(x)}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta-x)^2\right]}\,dx - \frac{1}{4}\int_{-\infty}^{+\infty}\frac{g_{l0}(\vartheta_1)\,d\vartheta_1}{\left[\left(\frac{\pi}{2}\right)^2+(\vartheta_1-\vartheta)^2\right]}$$

Thus

$$\frac{df_{u1}}{d\xi} = -\frac{\beta}{2\pi U}\int_{-\infty}^{+\infty} \frac{u_{i0}(x)}{(\xi - x)}\,dx - \frac{1}{2}\frac{df_{u0}}{d\xi}$$

$$+ \frac{\beta}{2\pi U}\int_{-\infty}^{+\infty} \frac{(\xi - x)\, v_{i0}(x)\, dx}{[(\beta h)^2 + (\xi - x)^2]}$$

$$- \frac{\beta h}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{[(\beta h)^2 + (\xi - x)^2]}\,\frac{df_{i0}}{dx}\,dx \quad -- (E1)$$

But

$$f_{u1}(x') = \int_{-L}^{x'} \frac{df_{u1}}{d\xi}\,d\xi$$

where L is the semi-length of the flexible tunnel section. Therefore equation (E1) can be integrated to give :-

$$f_{u1}(x') = -\frac{\beta}{2\pi U}\int_{-L}^{x'} d\xi \int_{-\infty}^{+\infty} \frac{u_{i0}(x)}{(\xi - x)}\,dx - \frac{1}{2} f_{u0}(x')$$

$$+ \frac{\beta}{2\pi U}\int_{-L}^{x'} d\xi \int_{-\infty}^{+\infty} \frac{(\xi - x)\, v_{i0}(x)}{[(\beta h)^2 + (\xi - x)^2]}\,dx$$

$$- \frac{\beta h}{2\pi}\int_{-L}^{x'} d\xi \int_{-\infty}^{+\infty} \frac{1}{[(\beta h)^2 + (\xi - x)^2]}\,\frac{df_{i0}}{dx}\,dx$$

$$= -\frac{\beta}{2\pi U} \int_{-L}^{+L} u_{io}(x) \ln\left|\frac{x-x'}{x+L}\right| dx - \frac{1}{2} f_{uo}(x')$$

$$+ \frac{\beta}{4\pi U} \int_{-L}^{+L} V_{io}(x) \ln\left[\frac{(\beta h)^2 + (x-x')^2}{(\beta h)^2 + (x+L)^2}\right] dx$$

$$- \frac{\beta h}{2\pi} \int_{-L}^{+L} \frac{1}{\left[(\beta h)^2 + (x-x')^2\right]} f_{io}(x)\, dx$$

But $f_u(x) = f_{ui}(x) + f_{uo}(x)$.
Hence, with interchange of variables, we have :-

$$f_u(x) = \frac{1}{2} f_{uo}(x)$$

$$- \frac{\beta}{2\pi U} \int_{-L}^{+L} u_{io}(x') \ln\left|\frac{x'-x}{x'+L}\right| dx'$$

$$+ \frac{\beta}{4\pi U} \int_{-L}^{+L} V_{io}(x') \ln\left[\frac{(\beta h)^2 + (x'-x)^2}{(\beta h)^2 + (x'+L)^2}\right] dx'$$

$$- \frac{\beta h}{2\pi} \int_{-L}^{+L} \frac{1}{\left[(\beta h)^2 + (x'-x)^2\right]} f_{io}(x')\, dx'$$

— (E2)

By a similar process, equation (D27) can be reduced to :-

$$f_1(x) = \frac{1}{2} f_{1o}(x)$$

$$+ \frac{\beta}{2\pi U} \int_{-L}^{+L} V_{io}(x') \left| \ln \left| \frac{x'-x}{x'+L} \right| \right| dx'$$

$$- \frac{\beta}{4\pi U} \int_{-L}^{+L} u_{io}(x') \left| \ln \left[\frac{(\beta h)^2 + (x'-x)^2}{(\beta h)^2 + (x'+L)^2} \right] \right| dx'$$

$$- \frac{\beta h}{2\pi} \int_{-L}^{+L} \frac{1}{\left[(\beta h)^2 + (x'-x)^2 \right]} f_{uo}(x') dx'$$

— (E3)

Equations (E2) and (E3) are in a form suitable for programming.

Care must be taken with the principal value integrals as follows :-

(1) analytic integration must be used in the region of $x' \simeq x$

(2) the values of u_{io} and V_{io} must be exactly zero in the region of $x' = -L$.

NASA National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-181661		

4. Title and Subtitle	5. Report Date
An Exact Strategy for Flexible Wall Adjustments in Two-Dimensional Compressible Flow	March 1981
	6. Performing Organization Code

7. Author(s)	8. Performing Organization Report No.
Michael Judd	
	10. Work Unit No. 505-61-01-02

9. Performing Organization Name and Address	11. Contract or Grant No.
University of Southampton Department of Aeronautics and Astronautics SO9 5NH Hampshire ENGLAND	NSG-7172
	13. Type of Report and Period Covered: Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225	14. Sponsoring Agency Code

15. Supplementary Notes

Technical Monitor: Charles L. Ladson

16. Abstract

A strategy is presented for the adjustment of the flexible top and bottom walls of a two-dimensional (2-D) Adaptive Wall Test Section (AWTS). This adjustment is part of the wall adaptation process intended to eliminate top and bottom wall interference at its source. This strategy is exact within the limits of the associated linearized compressible flow theory. In this strategy, the effect of one flexible wall movement on the other is analytically modeled. In the well tested 2-D strategy previously proposed by Judd, this effect was accounted for by empirical factors. The author anticipates that this improvement over the old 2-D strategy should eliminate or further reduce the number of iterations necessary in the wall adaptation process. Appendices contain detailed mathematical derivations of the equations forming the analytical link between the adjustment of the two flexible walls.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Adaptive walls 2-D wall adjustment strategy 2-D wall interference Flexible walled wind tunnels	Unclassified - Unlimited Subject Category - 02

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	35	